Active Skin for Turbulent Drag Reduction

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ABSTRACT

Drag reduction for aerial and underwater vehicles has a range of positive ramifications: reduced fuel consumption, larger operational range, greater endurance and higher achievable speeds. This work capitalizes on recent developments in Direct Numerical Simulations (DNS) studies on turbulent drag reduction techniques and active material based actuation to develop an “active” skin for turbulent drag reduction. The skin operation principle is based on computational evidence from DNS studies indicating that spanwise traveling surface waves of the right amplitude, wavelength and frequency can result in significant turbulent drag reduction. Such traveling waves can be induced in the skin via active-material actuation. The flow control technique pursued is “micro” in the sense that only micro-scale wave amplitudes (order of 30 µm) and energy inputs are expected to produce significant benefits. In this work, a generalized actuation principle that can generate a traveling surface wave is proposed and analyzed. Two actuation schemes, that are approximate implementations of this generalized actuation principle, are considered. Treating these actuation schemes as design paradigms, several skin designs are developed. The feasibility of the different actuation possibilities, such as Shape Memory Alloys and Piezoelectric material based actuators, and the relative merits of the different skin designs are discussed. The feasibility studies include analytical solutions based on geometric idealizations, that are further refined using quasi-static Finite Element Analysis (FEA). The dynamic response of the skin designs to complicated loading sequences, are investigated using dynamic finite element analysis. Rayleigh damping is used in the dynamic analysis to model the total damping present in the system.

Keywords: Turbulent Drag, Shape Memory Alloys, Piezoelectric materials, Actuator, Active Skin

1. INTRODUCTION

Modern turbulence control methods aimed at turbulent drag reduction have typically been developed based on the observation that any weakening of the streamwise vorticity results in a corresponding reduction in turbulent drag. To this end, the use of small grooves or riblets mounted on the wall surface has proved to be effective in partially suppressing turbulence and reducing the drag force by about 5% to 10% [\cite{1},\cite{2}]. However, to date none of the drag reduction techniques used in practice has been successful in eliminating near-wall streaks, the instability of which is considered responsible for sustaining the turbulence production cycle. In a recent work, a new fundamental mechanism was revealed, which could completely eliminate the near-wall streaks [\cite{3},\cite{4}]. In this method, streamwise vorticity is introduced in the flow in such a manner that, it helps regularize and stabilize the streaks and in many cases eventually eliminate the streaks altogether. This new method is based on the application of a traveling force wave that travels along the spanwise direction, acting within the viscous sublayer and with the force decaying exponentially away from the wall. In particular, the following traveling wave force was used:

\[ F_z = I e^{\frac{x}{A}} \sin(\frac{2\pi}{\lambda} z - \frac{2\pi}{T} t) \]  

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where $I$ is the amplitude of excitation, $\lambda$ is the wavelength (along the span), $T$ is the time period and $\Delta$ is the penetration depth ("z" refers to the spanwise direction; force acts along the same direction as the direction of the wave propagation).

A parametric study of the effect of the traveling wave force indicated that a maximum drag reduction (of over 30%) was achieved when the non-dimensional wavelength was around 1000 wall units and when the non-dimensional penetration depth was around 5 wall units (which corresponds to the thickness of the viscous sublayer). This relatively small requirement for the penetration depth is the basis for the assumption that an "active" skin that would generate a traveling surface wave, will in effect result in generating a traveling force wave. The skin could be attached to the surface of an aircraft wing, for example, and be deformed to the shape of the required wave using actuators made of active materials.

The flow control technique envisaged is an active flow control technique (which involves an energy input into the system) as compared to a passive flow control technique such as the use of riblets. Moreover, the skin is designed to be actuated using active material actuators. For these reasons the skin is referred to as an “active” skin.

By converting the non-dimensional optimal wave parameters to dimensional values for certain practical applications, it is found that, for typical Reynolds numbers encountered during flight conditions in Unmanned Aerial Vehicles (UAVs), the optimal parameters of a surface wave would be a wavelength of around 25 mm, an amplitude (which corresponds to the penetration depth) of around 30 $\mu$m and a frequency in the range of a few hundred hertz. For Remotely Operated Vehicles (ROVs), the optimal frequency would drop down by an order of magnitude, whereas for Submarines the wave amplitudes and wavelengths would drop by an order of magnitude while the frequencies would increase to the kHz range. Since the optimum wave amplitudes are in the micron range it is expected that the energy inputs required to sustain the traveling wave would be significantly lower than the gain achieved by drag reduction. Active material actuators, based on Shape Memory Alloys (SMAs) and Piezoelectric materials, provide, in a complimentary manner, a wide range of actuation capabilities that can satisfy the actuation requirements for the active skin for a wide range of applications.

### 2. PRINCIPLE OF ACTIVE TRAVELING WAVE SKIN

In this section, a theoretical basis for the development of a practical “Active Skin” is presented. The key assumption in the development of the active skin is that, a traveling surface wave, will in turn result in a self-similar transverse traveling force wave that acts on the fluid next to the skin. Here, by “self-similar”, it is meant that the response is similar in shape to the loading function. Hence, the requirement for the active skin is that, the skin should deform to take the shape of a traveling sine wave, that is similar in form to the one given in equation (1).

Theoretically, the simplest method of generating such a traveling sine wave on the surface of a thin rectangular plate (which is the skin), would be by subjecting the skin to a distributed vertical loading, which also resembles the required traveling sine wave. This claim is proven in this section, following a procedure akin to that used in the derivation of the forced response of damped single-degree-of-freedom systems. Figure 1 depicts the applied distributed load on the skin and the accompanying deflection in the skin.

![Figure 1. Forcing function and the accompanying self-similar deflection.](image)

Since it is required that the bending in the skin be cylindrical in nature, the loading would to be a constant in the "width" direction (normal to the page) of the thin rectangular sheet. The governing partial differential equation (PDE) for cylindrical bending in thin rectangular plates (linear elastic and isotropic material) undergoing small strains (in the absence of an elastic foundation) is given by the following equation [[5], [6]]:
\[
\frac{\partial^4 y}{\partial x^4} = \frac{1}{D} \left[ q - \rho h \frac{\partial^2 y}{\partial t^2} + \frac{\rho h^3}{12} \frac{\partial^4 y}{\partial x^2 \partial t^2} - \frac{1}{1-\nu} \frac{\partial^2 M_T}{\partial x^2} \right]
\]  
(2)

where \( D = Eh^3/(12(1-\nu^2)) \); \( y \) is the deflection of the plate; \( E \), the Young’s Modulus; \( \nu \), the Poisson Ratio; \( h \), the thickness of the plate; \( q \), the applied vertical force per unit area and \( M_T \) is the applied external moment per unit depth of the plate (due to a thermal gradient across the cross section of the plate).

It is clear that the second derivative of the moment in (2) would have the same form as the moment itself, albeit with a phase difference of 180°, if the applied moment were a harmonic of the form given in (1). This implies that application of loads and moments are equivalent. Thus, a traveling sine wave can be generated on the surface of the active skin by means of a distributed moment, which takes a shape similar (though with a phase difference) to that of the required surface wave. Ignoring the contribution from the rotational inertia term, and under the condition of no externally applied thermal moment, the partial differential equation reduces to:

\[
D \frac{\partial^4 y}{\partial x^4} + \rho h \frac{\partial^2 y}{\partial t^2} = q
\]  
(3)

The loading function \( q(x) \), can be split into three contributing functions, \( f_l(x) \), \( f_d(x) \) and \( f_i(x) \). While the first function represents the actual externally applied vertical loading per unit area, the second function represents the viscous reaction forces (per unit area) exerted by the fluid back on the skin due to the motion of the skin. There are also inertial forces exerted on the skin by the fluid, on account of the acceleration of the skin and the subsequent transfer of momentum to the fluid. These forces are taken into account by the third function. The three forces considered for this analysis can therefore be written as:

\[
q(x) = f_l(x) + f_d(x) + f_i(x)
\]  
(4)

\[
f_l(x) = a \sin(kx - \omega t)
\]  
(5)

\[
f_d(x) = -c \frac{\partial y}{\partial t}
\]  
(6)

\[
f_i(x) = -b \frac{\partial^2 y}{\partial t^2}
\]  
(7)

Here, \( L \) and \( T \) are the wavelength and time period, respectively, of the traveling wave-like loading function, and are identical to \( \lambda \) and \( T \) used in (1). \( a \), \( c \), \( b \) are respectively the constants associated with the three forcing functions, \( f_l(x) \), \( f_d(x) \) and \( f_i(x) \). The expressions (6) and (7) are an extrapolation from the behavior found in linear single degree of freedom systems. The damping force in such systems is proportional to the velocity of the mass, and the inertial force is proportional to its acceleration. It should however be noted that, considering \( b \) and \( c \) to be constant with respect to \( x \) is a significant assumption which is as yet uncorroborated experimentally. Rewriting (3) using (4), (5), (6) and (7) gives:

\[
D \frac{\partial^4 y}{\partial x^4} + \frac{\rho h}{\rho_{\text{eff}}} \frac{\partial^2 y}{\partial t^2} + \frac{\rho h}{\rho_{\text{eff}}^2} \frac{\partial^4 y}{\partial x^2 \partial t^2} = A \sin(kx - \omega t)
\]  
(8)

\[
\rho_{\text{eff}} = \rho \left[ 1 + \frac{b}{\rho h} \right]
\]  
(9)

\[
A = \frac{a}{\rho_{\text{eff}} h}
\]  
(10)

Here, \( \rho_{\text{eff}} \) is the effective density and is a factor that represents the combined inertial effects of both the skin and the fluid. Since the complimentary solution of this PDE would decay with time, only the particular integral needs to be considered when the steady state response is of interest ([7]). Consider an infinitely long skin. Any two points on this infinitely long skin, separated by a distance \( L \), are identical in all respects since the two points undergo identical loading. Hence, their response must also be identical. Therefore, the boundary conditions for this problem would be:
Consider a possible solution of the form given in (12):

\[ y(x, t) = Y(k, \omega) \sin(kx - \omega t - \phi) \]  

This form clearly satisfies the boundary conditions given in (11). Here \( Y(k, \omega) \) represents the amplitude of the skin’s response to the excitation, while \( \phi \) represents the phase lag between the excitation and the response. This possible solution is clearly self-similar to the original excitation on the skin. Changing over to complex notation and substituting (12) into (8) yields, upon further simplification, the following solution for the response of the skin:

\[ y(x, t) = \frac{a}{\sqrt{\left( Dk^4 - \rho_{\text{eff}} \omega^2 \right)^2 + (c \omega)^2}} \]

\[ \phi = \tan^{-1} \left[ \frac{c \omega}{Dk^4 - \rho_{\text{eff}} \omega^2} \right] \]

The quasi-static response amplitude is derived from (13) by letting the forcing frequency be zero, and is given as:

\[ Y(k, \omega)_{\text{quasistatic}} = \frac{a}{Dk^4} \]

The condition of resonance corresponds to the choice of \( \omega \) that makes \( Y(k, \omega) \) infinite, in the absence of damping. Thus, the first undamped natural forcing frequency, \( \omega_n \), is given as:

\[ \omega_n = k^2 \sqrt{\frac{D}{\rho h}} \]

Note here that the effective density term has been replaced with simply the density of the skin material. This is because the expression given in (15) represents the undamped natural frequency, which is the natural frequency of the skin in the absence of a fluid medium. Thus the application of a loading function of the form given by (5) results in a deflection response that is self-similar to the loading function, but phase shifted by a certain amount as given in (13). Since in linear PDEs uniqueness of solution is guaranteed, the solution given by (13) is the only solution for the PDE.

However, the conclusion of self-similar response cannot be extended to an arbitrarily shaped traveling wave-like loading that is periodic in space and time. Such a loading can in general be represented in the following form:

\[ f(x, t) = f(kx - \omega t + nP) \]

\[ n = 0, \pm 1, \pm 2, \pm 3... \]

The \( (kx-\omega t) \) term indicates the traveling nature of the function and the \( nP \) term indicates its periodic behavior. \( P \) may be viewed as a “generalized period” of the function, and it follows directly from this and the earlier definitions that:

\[ P = kL \]

\[ P = \omega T \]
\[ f(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(n\omega_0 z) + b_n \sin(n\omega_0 z)] \]
\[ z = kx - \omega t \]
\[ \omega_0 = \frac{2\pi}{P} \]

Since the system is linear, the total system response is the sum of the responses to each of the individual terms in the Fourier expansion. Using the result derived in (13), the total system response can be written as:

\[ y(x, t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} Y(nz) \left[ a_n \cos(n\omega_0 z - \phi_n) + b_n \sin(n\omega_0 z - \phi_n) \right] \]

\[ Y(nz) = \frac{a}{\sqrt{Dk^4 - \rho \eta^2 h^2 \omega^2 + (c_n \omega)^2}} \]
\[ \phi_n = \tan^{-1} \left[ \frac{c_n \omega}{Dk^4 - \rho \eta^2 h^2 \omega^2} \right] \]

The sum of this infinite series of response contributions, from the harmonics that make up the frequency spectrum of the forcing function, will in general, not be self-similar to the forcing function itself. However, the response would still be periodic in space and time with the same period as the forcing function. For example, one of the harmonics that make up the forcing function might be very close to a resonance frequency of the skin. In this case, the response of the skin would be dominated by the response contribution from that specific harmonic. Only for cases where the response required is a harmonic (and not just a periodic function), will this technique produce the desired result, since the Fourier series expansion will simply be the function itself.

To sum up, the closed form solution for a generalized loading was obtained and it was seen that if the loading function is a traveling wave in the form of a harmonic, then the response would be self-similar to the loading, but with a phase lag.

3. ACTUATION SCHEMES FOR ACTIVE TRAVELING WAVE SKIN

A traveling surface wave, that is periodic in space and time, requires that every point on the surface undergo an identical response, which is merely phase shifted from the response of any other neighboring point. Moreover, the phase shift in the response between the two points must be a constant that depends solely on the distance between the two points.

However, in reality, the mechanical loading of the skin can never be performed in a truly continuous manner. Use of a discrete number of actuators, automatically implies that a discrete number of points that are directly connected to the actuators, would undergo a loading that is different from the rest of the regions on the skin.

The difference in response (other than the phase difference) between any two given points on the skin, that is caused by this specificity of points of loading, might be severe or negligible, depending on the distribution density of the actuators used. It is generally expected that, greater the density of actuators used, closer would be the approximation of a continuous distribution of forces/moments. The loading produced by a discrete number of actuators in an active skin implementation would, in general, resemble a functional form given by:

\[ F(x, t) = g(x + mL) f(kx - \omega t + nP) \]
\[ n = 0, \pm 1, \pm 2, \pm 3... \]
\[ m = 0, \pm 1, \pm 2, \pm 3... \]

Here, \(f(kx - \omega t - nP)\) retains the same meaning as defined in (16) and represents the traveling wave component of the loading. However, there is a periodic function, \(g(x + mL)\), with period equal to the wavelength of the traveling wave,
that is superimposed onto this traveling wave form. Equation (20) indicates that the loading experienced by each point is periodic and phase shifted by a constant phase difference with respect to other points (the phase shift being solely a function of the distance between the points). However, the amplitude of this loading does depend on the specific location, but again in a periodic manner given by \( g(x + mL) \). It should be noted that the functional form given in (20) is not a standing wave, since there is a constant phase difference between the loading cycle of any two neighboring points.

Hence, in the actual implementation of the active skin, the response of the skin will be in most cases only an approximate traveling sine wave. On account of this fact, the DNS studies were also extended to include shapes that are approximate versions of traveling sine waves. The DNS studies indicated that even approximate versions of traveling sine waves produced nearly the same effect. Here again, the implicit assumption is that the deflections and the resultant forces on the fluid would be self-similar to the loading function. This assumption is definitely justifiable if the forcing frequencies are small compared to the natural frequency of the skin.

In this section, two possible actuation schemes that could act as design paradigms for active skin designs are discussed. Since these actuation schemes are based on the use of a discrete number of actuators, they only produce approximate traveling sine waves. However, as mentioned earlier, a dense distribution of actuators is expected to approximate closely a continuous distribution of loads.

The first actuation scheme is depicted as a free body diagram in Figure 2. Consider a long thin plate (skin) subjected to equally spaced external moments that are equal in value but alternate in sign. This loading pattern generates a static wavy profile. By simultaneously shifting the points of application of the moments in one direction, the static profile can be made to “travel” in that particular direction, resulting in a traveling surface wave. The moments can be realized either by the application of equal and opposite forces (couple) at those points or by unbalanced lateral forces separated from the skin by a lever arm.

The second actuation scheme utilizes equally spaced vertical external forces (discretely located) along the length of the skin. Each wavelength of the skin is loaded by eight actuators with each successive actuator lagging behind the earlier one by a phase difference of 45°. The amplitudes and the directions of the forces generated by these actuators vary periodically and match “pointwise” that of a continuous traveling sine wave form as shown in Figure 3.

In this second actuation scheme, it is possible to view two sets of equal and opposite forces (which are separated by a distance of half a wavelength) to constitute a couple. Thus even in this approach, there is essentially a distribution of moments along the length of the skin. However, in this scheme the points of application of the moments remain constant, whereas the values of the actuating moments vary in a periodic manner. This situation is the exact opposite of the first actuation scheme in which, the actual values of the applied moments are constant, but the points of
application of these moments change in a periodic manner. A traveling wave profile can be generated by simultaneously varying all the forces in a periodic manner. All the forces would oscillate with the same frequency and the same amplitude. However, each successive force would lag the previous force by a constant phase difference of 45°. In other words this would be a discrete implementation of a sinusoidal traveling wave form loading. Since the deflections are expected to be self-similar to the forces (atleast for actuation at frequencies much smaller than the natural frequency of the skin), the net effect would be a traveling wave.

These two actuation schemes are clearly examples of loading as described in (20). Furthermore, these two schemes also indicate the equivalence of forces and moments, which was noted earlier in section 1. Both the schemes are capable of producing an approximate traveling sine wave on the surface of the active skin.

4. THEORETICAL ANALYSIS OF THE ACTUATION SCHEMES

Theoretical analysis of the two actuation schemes mentioned in the earlier section is made difficult due to two reasons. Firstly, the forcing function that represents the loading in the two actuation schemes takes up the form given in (20), which involves the product of two different periodic functions. This form of loading does not yield easily closed form solutions for the governing PDE, when compared to loading of the form given in (5), which is a simple harmonic. Secondly, and more importantly, the specific form of the forcing functions in these two schemes is quite complicated. This is particularly the case with the Moment based actuation scheme, which involves points of singularity, as the point of application of moments change abruptly from one location to another. Because of the above mentioned difficulties, instead of attempting of solve the governing PDE analytically in a dynamic setting for the two actuation schemes, a quasi-static and an eigenvalue analysis are performed on the skin, to obtain a qualitative picture of the response of the skin.

Moreover, the representation of the skin as a thin sheet is only an idealization. In reality, the skin would be quite complicated in design so as to accommodate the actuator mechanism. Again, it would be very difficult, if not impossible, to consider, analytically, all the design details, while solving for the system response. Hence, Finite Element Analysis (FEA) is used to further refine the theoretical results in the latter sections, through analysis of the actual skin designs.

4.1. Reduced Periodic Element

To simplify the analysis, as was done earlier, it is possible to utilize the inherent periodicity in loading and boundary conditions of the active skin, to reduce the analysis domain to a Periodic Element. The Periodic Element can be defined as the smallest repeating unit of the active skin, the behavior of which would completely describe the behavior of the entire skin. The analytical expressions for the deflection amplitude and the natural frequency of the Periodic Element can be obtained by solving the general expressions that describe cylindrical bending in plates (equation (2)). Utilizing the periodicity assumption, the Periodic Element for the two actuating schemes shown in Figure 2 and Figure 3 can be obtained as the section of skin that is one wavelength long. Utilizing the symmetry in bending, the Periodic Element can be further reduced to a section of the skin that is half a wavelength long (Figure 4).

![Figure 4. Reduced Periodic Elements for the two actuation schemes.](image-url)
The appropriate boundary conditions for the Reduced Periodic Element would therefore be

- Symmetry conditions at the ends
- Zero deflection for the point in the middle of the Reduced Periodic Element

\( M_a \) and \( M_b \) are the bending moments at the boundaries of the Reduced Periodic Element that need to be solved for and are not the externally applied moments. In the free body diagram of the Reduced Periodic Element for the force based actuation technique, the applied external forces at the ends are half the value of the actual forces at these points, since the effects of these forces are equally shared by two adjacent Reduced Periodic Elements.

### 4.2. Analytical Derivations

The Moment Area method developed for beam bending \([8]\) can be extended to cylindrical bending of linear elastic plates undergoing small deformations, to solve for the quasistatic deflection amplitudes. The moment area method gives the tangential deflection of any two points along the length of the plate. Since the tangents at the two ends of the Unitcell are horizontal, the tangential deflection would simply be twice the required deflection amplitude. The deflection amplitude is estimated using the following relation in this method:

\[
\Delta = \frac{1}{2} \int_{A}^{B} M_{xx} x \, dx
\]  

(21)

Here \( M_{xx} \) is the resulting bending moment in the plate due to the loads and moments acting on it, and the integration is carried out along the length of the Reduced Periodic Element. Based on the symmetry between the two ends of the Reduced Periodic Element, it can be stated that the bending moments at the two ends, \( M_a \) and \( M_b \), are identical. Solving for the moment and the shear force distribution in the Reduced Periodic Element and using the moment area method, the quasi-static deflection amplitudes for the two actuation schemes are obtained as:

\[
\Delta_1 = \frac{3F_1b l^2 (1-\nu^2)}{4Eh^3 d}
\]  

(22)

\[
\Delta_2 = \frac{(16+11\sqrt{2})F_2 l^3 (1-\nu^2)}{64Eh^3 d}
\]  

(23)

where: \( \Delta_1 \) and \( \Delta_2 \) are the deflection amplitudes for the first and second schemes of actuation respectively; \( l \) is the length of the Reduced Periodic Element; \( d \) is the width of the Reduced Periodic Element; \( F_1 \) and \( b \) are the load and the load arm through which the external moment is generated (i.e. \( M = F_1 b \)) for the moment based actuation and \( F_2 \) is the force amplitude for the force based actuation.

For the case of free vibrations, ignoring the contribution from the rotational inertia term, the partial differential equation given in (2) simplifies to \([9]\):

\[
\frac{\partial^4 y}{\partial x^4} = -\frac{\rho h \partial^2 y}{D \partial^2 t^2}
\]  

(24)

The boundary conditions for this eigenvalue problem are that the slopes at the edges of the Reduced Periodic Element are zero and that the point of application of the external moment is both a point of inflection and a point of zero displacement. Solving for these boundary conditions, the expression for the \( n^{th} \) natural frequency (for both actuation schemes) is obtained as:

\[
f_n = \frac{\pi n^2 h}{2l^2} \sqrt{\frac{E}{12\rho(1-\nu^2)}}
\]  

(25)

where \( f_n \) is the natural frequency and \( n \) is the eigenmode. The above expression for the natural frequencies is identical to that derived from (15), which is as expected.
4.3. Comparison of Work Efficiencies

The work done to generate the same deflection amplitudes, in the two actuation schemes, can be used to compare the efficiencies of the two schemes. Assuming actuation under quasi-static conditions, the work done can be evaluated as the integral of the strain energy stored in the entire volume of the plate and is given in the case of moment based actuation scheme by the expression:

\[
W_1 = \frac{3h^2l(l-\nu^2)}{2Eh^3d}F_1^2
\]  

(26)

Alternatively, this expression can be obtained as half the product of the external moment and the angle of rotation at the point of application of the moment. The work done for the force based actuation scheme is:

\[
W_2 = \frac{(16+11\sqrt{2})l^3}{64Eh^2d}(1-\nu^2)F_2^2
\]  

(27)

In a similar manner this same expression can be obtained by considering the sum of the individual contributions from each of the external forces (the points of application of the forces deflect in the direction of the forces). To generate the same deflections, the ratio of the forces, needed for the two actuation schemes, is given by:

\[
\frac{F_1}{F_2} = \frac{(16+11\sqrt{2})l}{48b}
\]  

(28)

Using this relationship, the ratio of the work done in the two actuation schemes, while generating identical displacement amplitudes (i.e. the work efficiency) is evaluated as:

\[
\frac{W_1}{W_2} = 1.315
\]  

(29)

Note that the ratio of the work inputs is independent of the dimensions of the plate. Therefore, it can be concluded that from a structural point of view, the force based actuation scheme is more efficient than the moment based actuation scheme. This is because the moment based actuation scheme generates larger curvature in the plate (and thus stores greater energy) while producing a certain displacement, than the force based actuation scheme. However, when it comes to actual implementation of these two concepts, many other factors finally would determine which design is better suited for the application at hand and these would be discussed later.

4.4. Significance of Analytical Studies

The significance of identifying the relationship between the deflection amplitude/natural frequency and the skin dimensions lies in the fact that they greatly aid in estimating approximate model dimensions \((L \& h)\) based on performance requirements without actually attempting multiple iterations of finite element analysis. For example, it is obvious from the proportionalities that decreasing skin thickness \((h)\) is more effective in increasing amplitude (or conversely in reducing force required for the same amplitude) than increasing length \((L)\).

Consider a sample case that uses the moment based actuation scheme: \(L = 25\text{mm}, h=0.5\text{mm}\) with the material being Aluminum of \(E=70\text{Gpa}, \nu =0.345\) and \(\rho =2600 \text{ kgr/m}^3\); the force applied is \(F_1=1\text{N}\); the load arm is \(b = 3\text{mm}\). For these parameters, the amplitude would be \(41.29\mu m\) and the natural frequency would be equal to \(8021.6\text{ Hz}\). Reducing the thickness by half, i.e. \(h = 0.25\text{mm}\), results in a deflection amplitude of \(330.34\mu m\) which is an order of magnitude greater than the response with \(h = 0.5\text{mm}\). Conversely a force of just \(F_1 = 0.1388\text{N}\) is sufficient in obtaining the earlier response of \(41.29\mu m\) in this case. It is this kind of insight into the system that is very useful during design iterations.

The value of natural frequency of the skin can also prove to be crucial. If the actuation frequency were to be made equal to the first natural frequency of the skin, this would set the skin into resonance. Since the eigenmode shape corresponding to the first natural frequency is identical to the deflection pattern desired, this resonance phenomenon would amplify the deflection pattern. Consequently, it is expected that this resonance would bring down the force requirements and hence the actuation costs.

It can be mentioned generally that a reduction in thickness or an increase in length is beneficial in two ways. It not only results in an increase in the deflection amplitude for the same force but also in the reduction in the value of the
first natural frequency. The latter effect is significant to our efforts in utilizing resonance to reduce actuation costs. By reducing the frequency from around 8kHz ($h=0.5\text{mm}$) to a value around 4kHz ($h=0.25\text{mm}$), it makes it possible to find realistic actuation strategies that can deliver forces at resonance frequencies. However, it should be noted that while the thickness of the skin is a parameter that can be varied freely, the wavelength is a parameter that is specified by the flow physics and its range of variation is therefore limited for a given application.

5. SKIN DESIGNS AND ACTUATION STRATEGIES

5.1. SMA Actuator Based Active Skin

Three different active skin designs that would be capable of creating a traveling wave profile using either of the actuation schemes have been considered. Figure 5 presents a cross section and a top view of the first skin design in its non-actuated state. This design works on the principle of moment based actuation. The moments that bend the skin are created by lateral forces that act on the skin through “legs” that are attached to the top surface of the skin. The legs can slide (left and right) with respect to the bottom surface (which is the surface attached to the vehicle) while the upper surface is exposed to the flow. The “legs” are actuated in a manner that induces rotation of the legs, which in turn results in a deformation of the top surface. When the legs are actuated in a coordinated manner, a wavy deformation pattern on the upper surface is generated. This coordinated leg actuation/rotation will be described shortly.

Shape Memory Alloys (SMAs) are a class of alloys that undergo a change in crystal structure from a cubic austenitic phase to a number of martensitic variants either upon cooling or on application of stress (or a combination of both) to produce large changes in shape [[10]-[26]]. Upon either heating or removal of the applied stress (or a combination of both), the reverse phase transformation occurs that results in the recovery of the original shape. By suitably constraining the shape memory alloy during this shape recovery, it is possible to use it as an actuator capable of producing large forces (stress of up to 350 MPa) and displacements (strains up to 8 %) [[26]].

![Figure 5. Cross section and top view of SMA actuated active skin.](image)

![Figure 6. Resulting waveform after actuating SMA sections between legs 1 and 5 and between legs 9 and 13 (y-dimensions have been magnified to illustrate the principle).](image)
In the above design, an SMA wire runs through the “legs” as shown in Figure 5, through small holes on their sidewalls. The SMA wire runs in the spanwise direction, while the major dimension of the “legs” is along the streamwise direction. Within each “leg” a circular flat disk is attached to the SMA wire, with its diameter significantly larger than the diameter of the holes on the sidewalls of the “legs”. Each SMA-disk joint is electrically connected to the electrical control circuit, and is powered independently. When a voltage difference is applied between the leftmost (“joint 1”) and the rightmost SMA-disk (“joint 5”) joints in Figure 5, joule heating would occur in the SMA wire that lies between the two disc joints. This would result in a negative strain in the SMA (on account of the wire contracting) that will cause the disks of joints 1 and 5 to contact the walls of legs 1 and 5, thus transferring to them the load generated by the SMA.

As shown in Figure 6 in an exaggerated fashion, as the SMA sections between legs 1 and 5 and between legs 9 and 13 contract, the SMA section between legs 5 and 9 will have to elongate/strain accordingly. Therefore, the SMA sections between 1 and 5 and 9 and 13 will have to produce enough force not only to deform the upper skin but also to strain the SMA section between legs 5 and 9. This requirement is typical in antagonistic SMA actuators and presents no problem, since the sections between 1 and 5 and 9 and 13 are austenitic and have a much larger stiffness (2 to 3 times higher) than the section between 5 and 9, which is in the martensitic phase. Another point to note is that the bending caused by the actuation would not be exactly uniform all along the streamwise direction on account of the fact that the loading takes place at discrete points. However, it is assumed that the deviation would be negligible if the SMA actuators were placed in relatively small intervals in the streamwise direction.

5.2. Piezoelectric C-Block Actuator Based Active Skin

SMAs are one of the many possibilities discussed as possible mechanisms for actuation. The same structural design (i.e. the same actuation concept) can be implemented using Piezoelectric C-block actuators [[27]-[29]] for high Reynolds numbers applications, where the required actuation frequencies exceed the bandwidth SMAs are capable of.

Piezoelectric materials exhibit a change in shape upon the application of an electric field. This property is used in creating actuators that are capable of generating moderate forces, at very high actuation frequencies, but at relatively small displacements. One of the techniques used to magnify the displacement produced by the piezoelectric actuators is the C-Block configuration. A C-Block actuator has a semi-circular composite structure consisting of two piezoelectric layers separated by a non-piezoelectric substrate. By suitably applying electric fields, it is possible to generate differential strains in the different layers that result in a net bending motion of the composite structure. The individual C-block actuator, which is configured in a semi-circular shape, can then be aligned in series or in parallel so as to multiply the deflection or force output to that required by the application at hand, while simultaneously fitting within the space constraints. Figure 7 illustrates the principles of piezoelectric actuation of the active skin in the second design.

When electrical voltage is properly applied to the semi-circular, C-block piezoelectric actuators, positioned between the first and the fifth “legs” the net effect is identical to that described in the SMA actuated design. The first and the fifth joints would feel the net force of the four intervening C-Block actuators as these joints will be the ones in which the discs will contact the legs first. The resultant forces causes “leg” rotation and subsequently deflection of the skin. The concern of having antagonistic neighboring C-blocks can be avoided by applying a reverse polarity on the neighboring C-blocks, so as to make them bend outward to accommodate the deflections.

Figure 7. Piezoelectrically actuated active skin.
5.3. Piezoceramic Stack Actuator Based Active Skin

A third skin design is shown in Figure 8. This design works on the force based actuation scheme. In this design, the “legs” are replaced by linear Piezoceramic Stack Actuators ([30]) (PSAs), which actuate the skin in a direction along their axis. On actuation, the PSAs exert a force on the skin in a direction normal to the skin, causing it to bend. By varying the values of the forces (in the PSAs) along the length of the skin in a periodic fashion it is possible to achieve a static bending in the form of a sinusoidal wave. By varying the force applied by each PSA with time in a periodic fashion it would be possible to obtain a traveling wave. In effect, displacement of the skin would follow the loading of the PSAs. This claim will be shown to be valid in a later section on dynamic analysis.

6. STATIC FINITE ELEMENT ANALYSIS

To study the effect of the introduction of “legs” in the geometry of the model, a parametric study using Finite Element Analysis was performed for the SMA actuator based design, that incorporated the geometric details of the “legs” in the FE model. A Static Plane Strain analysis of the Reduced Periodic Element was carried out using the Finite Element solver ABAQUS. Quadratic Plane Strain elements were used to mesh the model. The mesh was successively refined until further refinement did not produce any changes in the result up to the 5th significant digit. The meshing corresponding to this accuracy had four quadratic plane strain elements in the thickness direction of the skin. The symmetry boundary condition implies that the shear force, slope and the axial deflection at the ends are zero. This was enforced by restricting the nodes along the edges to have zero horizontal displacement.

The deflection amplitudes from the FEA of the model (with “legs”) were compared with the results obtained from a geometry without “legs”. It should be noted that for a geometry without “legs”, both FEA and the theoretical analysis yield nearly identical results, since both approaches nearly capture all the details of the actual system. The parametric study revealed that the deflection amplitudes for a geometry with “legs”, deviated by greater amounts from the analytical results (i.e. results in the absence of “legs”), with either decreasing Length (L) or with decreasing Thickness (h). Both decreasing Length and decreasing Thickness represent instances in which the effect of the legs becomes more significant. Therefore, this trend is expected and is shown in Table 1.

Table 1. Comparison of analytical and FEA results for the deflection amplitude.

<table>
<thead>
<tr>
<th>Thickness h in mm</th>
<th>Length L in mm</th>
<th>Deflection Amplitude in µm (FEA)</th>
<th>Deflection Amplitude in µm (No “legs”)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>4.82</td>
<td>5.16</td>
<td>7.1</td>
</tr>
<tr>
<td>0.75</td>
<td>25</td>
<td>10.66</td>
<td>11.8</td>
<td>10.7</td>
</tr>
<tr>
<td>0.5</td>
<td>25</td>
<td>32.42</td>
<td>38.35</td>
<td>18.3</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>9.58</td>
<td>10.12</td>
<td>5.6</td>
</tr>
<tr>
<td>0.75</td>
<td>35</td>
<td>21.46</td>
<td>23.13</td>
<td>7.8</td>
</tr>
</tbody>
</table>
The material used for analysis was Aluminum of $E=70\text{GPa}$, $\nu=0.345$ and $\rho=2600 \text{ kg/m}$ and the moment arm was taken as $b=3\text{ mm}$. (Note: $L = \text{Wavelength of the traveling wave} = \text{twice the length of the Reduced Periodic Element} = 2l$)

<table>
<thead>
<tr>
<th>Thickness $h$ in mm</th>
<th>Length $L$ in mm</th>
<th>Natural Frequency in Hz (FEA)</th>
<th>Natural Frequency in Hz (No “legs”)</th>
<th>% Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>10816.2</td>
<td>16043.1</td>
<td>48.3</td>
</tr>
<tr>
<td>0.75</td>
<td>25</td>
<td>7663.9</td>
<td>12031.7</td>
<td>57.0</td>
</tr>
<tr>
<td>0.5</td>
<td>25</td>
<td>4668.2</td>
<td>8021.5</td>
<td>73.2</td>
</tr>
<tr>
<td>1</td>
<td>35</td>
<td>6209.5</td>
<td>8185.3</td>
<td>31.8</td>
</tr>
<tr>
<td>0.75</td>
<td>35</td>
<td>4416.8</td>
<td>6138.9</td>
<td>39.0</td>
</tr>
</tbody>
</table>

Table 2. Comparison of analytical and FEA results for the natural frequencies.

A similar trend of greater deviation from the theoretical values with both decreasing Length and decreasing Thickness was observed in the values of the natural frequencies. However, the deviations in this case were significantly larger than in the earlier case (Table 2).

This is because the legs contribute to a reduction in the values of the natural frequencies by providing greater inertia to the structure, without increasing the stiffness of the structure by a proportional extent. This claim is easily verified by performing FEA on two configurations of the skin that are identical in all respects except for the size of the legs. The skin dimensions used for this purpose were $L=25\text{mm}$ and $h=1\text{mm}$ with the material being Aluminum of $E=70\text{GPa}$, $\nu=0.345$ & $\rho=2600 \text{ kg/m}^3$. The two sizes of the legs used were $b=1\text{ mm}$ and $b=3\text{ mm}$ long. The results were then compared to the values obtained from the analytical expressions, which essentially is the case of the skin with no legs attached. The results are tabulated below (Table 3). It is clear that there is a considerable reduction in the value of the first natural frequency with an increase in the size of the legs. This confirms that the inertia factor introduced by the legs, far outweighs the accompanying increase in stiffness, resulting in a fairly large deviation from the results obtained for a geometry without legs. As the thickness of the skin drops, this effect of the legs becomes proportionally more significant.

<table>
<thead>
<tr>
<th>Cases</th>
<th>Natural Frequency in Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>No legs: $b=0$</td>
<td>16043.0</td>
</tr>
<tr>
<td>FEA: $b=1\text{mm}$</td>
<td>13982.5</td>
</tr>
<tr>
<td>FEA: $b=3\text{mm}$</td>
<td>10816.2</td>
</tr>
</tbody>
</table>

Table 3. Effect of height of legs on natural frequencies.

Thus it can generally be summarized that the presence of “legs” decreases the deflection amplitude by increasing the stiffness and decreases the natural frequency by increasing the rotational inertia in the system. However, the latter effect is more significant than the former.

Shown in Figure 9 is a typical contour plot of the axial stress distribution superimposed on the plot of the deformed Reduced Periodic Element (deformations magnified by 50 times). It is clear from the nature of the distribution that it essentially resembles the stress distribution in a beam subjected to a bending moment. Moreover, the leg in the center that is subjected to loading hardly suffers any bending. This assumption, which was implicit in the analytical calculations, wherein the actual applied force was transformed into an axial force and an equivalent moment, therefore stands justified. The first eigenmode of the Reduced Periodic Element is plotted in Figure 10 and shows that this mode shape is identical to the deflection pattern sought.
The periodicity assumption and the reduction of the analysis domain to a section of the skin half a wavelength long, was verified by performing FEA of the Periodic Element. The boundary conditions for this analysis were cyclic symmetry conditions at the ends, since the model was one entire wavelength long. This was done by enforcing the condition that the deflections of any two corresponding nodes at the two ends of the finite element model must be identical. The results obtained this way exactly matched the earlier results, thus confirming the validity of the periodicity assumption (Figure 11). A few sample cases were analyzed using FEA for the Piezoceramic stack actuator based design to verify that the FEA results matched the theoretical results. As mentioned earlier, for this geometry the assumptions made in the theoretical model are almost the same as in the FEA, and this matching of results is expected. A very small deviation in the results occurred due to the fact that the theoretical value is based on formulations derived from beam bending whereas the FEA results are obtained by solving a discretized plane strain problem.
7. FEASIBILITY OF ACTIVE MATERIAL BASED ACTUATION

Based on the earlier studies it was decided to consider, for the purpose of a feasibility study, a skin of dimensions $L=25\text{mm}$, $b=3\text{mm}$, $h=0.5\text{mm}$, that is made of Aluminum of $E=70\text{GPa}$ and $\nu =0.345$.

7.1. SMA Actuation Based Design

A SMA wire (actuator) was incorporated into the Finite Element analysis using the subroutine developed by this research group to implement the behavior of SMAs into the finite element solver ABAQUS. The subroutine numerically implements the unified thermomechanical constitutive model developed by Lagoudas et. al.\textsuperscript{[25]} which was further refined by Bo and Lagoudas\textsuperscript{[13]-[16]}.

The wire was positioned such that it joined the first and the fifth legs as shown in Figure 12 and essentially replaced the equal and opposite forces that were placed at these two points in the earlier analyses. A temperature change was used to actuate the wire to produce the deflections. The cross section area of the SMA wire used was $0.3\text{mm}^2$. The FE model used for this analysis was the periodic element (and not the reduced periodic element) since it was more convenient to define an actuator element in this geometry.

The analysis showed that a temperature change of $3^\circ\text{C}$ was sufficient in generating a deflection of amplitude $32.58\ \mu\text{m}$. This corresponded to a reduction of the martensitic volume fraction from the initial value of 1 to 0.8739 (martensitic volume fraction of zero indicates a complete transformation into the high temperature austenitic state). However it should be noted that this result is for a skin of width $d=1\text{mm}$. It would be difficult to place one SMA wire for every mm so as to obtain the necessary deflections across the depth of the skin. One wire which actuates a section of skin of width $d=5\text{mm}$ would be a more practical situation. Going by the analytical calculations it would be reasonable to expect that the force required when $d=5\text{mm}$ would be 5 times greater than that required for $d=1\text{mm}$.

It was found that a temperature change of $5^\circ\text{C}$ was capable of producing a deflection of $31.2\ \mu\text{m}$ when the width of the skin was taken as $5\text{mm}$. This also corresponded to a reduction in martensitic volume fraction to a value of 0.8677 and a strain of just 0.6%. This apparent lack of proportionality between the width of the skin and the temperature change is due to the non-linear response of the SMA wire to temperature change.

It is quite possible to heat and cool the SMA wire back and forth by a temperature change of $5^\circ\text{C}$ at relatively high frequencies. The efficiency of this cooling system can also be improved by using the cold air from the environment (in the case of aircraft). Even low flying UAVs such as the Predator operate at altitudes in the range of 10,000 feet. Taking the rate of drop in temperature with altitude as $6.5^\circ\text{C}/\text{km}$, the temperature of the atmosphere at such altitudes would be close to the freezing point of water. This cold air can be used as part of a heat exchange mechanism to cool down the SMA wires quite rapidly. The rate of cooling can be improved further if the operating temperatures of the SMA wires are kept high, since the rate of cooling would be proportional to the temperature differential between the wire and the air. By suitably adjusting the proportions of Ni and Ti in the SMA wire, it is possible to change the
austenitic and martensitic start and finish temperatures to considerably high values. SMA based actuation therefore seems to possess considerable promise in this drag reduction technique for low to moderate frequencies (order of 50Hz).

7.2. C-Block Actuator Based Design

The force required for these parameters is $F = 1\text{N}$ and the deflection of the point of application of the force is $35.28\ \mu\text{m}$ (in the legs). Going by the existing literature on C-Block actuators it is clear that these forces and displacements are well within the reach of C-Block actuators. However matching the geometric constraints of the active skin with the actuator might pose some challenges.

7.3 Piezoceramic Stack Actuator Based Design

The maximum force required for these parameters is $F = 0.31\text{N}$ and the corresponding deflection of the point of application = $30\ \mu\text{m}$. Commercially available stack actuators are well capable of delivering these requirements. However, one significant disadvantage is that the lengths of these actuators need to be in the order of 40 mm to deliver the required displacements. This can create complications in embedding these systems onto the surface of the aerial/underwater vehicle. Motion amplification mechanisms could resolve this problem but will introduce additional moving parts and design complications.

8. DYNAMIC FINITE ELEMENT ANALYSIS OF SMA ACTUATOR BASED DESIGN

Dynamic finite element analyses of the active skin designs were performed to evaluate the dynamic response of the skin to loading at different frequencies. Particularly, in the case of the SMA/C-Block actuator based design, the complicated series of loading and unloading steps is bound to introduce significant dynamic effects in the response of the skin, especially at high actuation frequencies. A complete evaluation of the feasibility of this design therefore requires a careful study of the skin response to different actuation frequencies. For completeness sake, the dynamic response of the Piezoceramic stack actuator based design is also analyzed for different actuation frequencies. However, it is reasoned that, the response of the skin in this design would be in general self-similar to the loading function itself (as described in section 2).

8.1. SMA/C-Block Actuator Based Design

“Multi-step” dynamic plane strain analysis was performed of the SMA/C-Block actuator based design using the finite element solver ABAQUS to study the effect of a series of loading steps that result in the generation of the traveling wave. The term “Multi-Step” here refers to the fact that analysis is carried through, for a number of loading steps corresponding to a duration greater than one period of the wave. The skin dimensions and the skin material used for this analysis are identical to those used in the static analysis performed during the feasibility studies ($L=25\text{mm}$, $h=0.5\text{mm}$ and $b=3\text{mm}$; skin made of aluminum with $E=70\text{GPa}$, $\nu=0.345$ and $\rho=2600\ \text{kg/m}$; force amplitude=1N).

On account of the inherent periodicity of the deflections in the skin, the domain of analysis is reduced to a section of the skin that is one wavelength long. This is achieved (as earlier) by enforcing the condition that the deflections of any two corresponding nodes at the two ends of the finite element model must be identical (Figure 13). The method of actuation (SMA or C-Block) is not important here, and hence equivalent forces are used in the analysis. In this case, it is not possible to further reduce the analysis domain to a section of the skin half a wavelength long, since there do not exist symmetries between the two edges of the Reduced Periodic Element as the loading proceeds from one leg to the other.
The geometric parameters of the legs are chosen such that, when the first loading step is underway between legs 1 and 5, the disks in the intermediate legs (legs 2,3 and 4) never come in contact with the respective leg walls, thus leaving these legs force-free. Simultaneously, legs 6,7,8 are force-free while leg 9 will be under loading due to the action of the actuator sections to its right. At the end of this first loading step (involving legs 1 and 5), the SMA section between legs 1 and 2 goes out of actuation as the point of application of voltage shifts abruptly to the disc on leg 2. Simultaneously, the SMA section between legs 5 and 6 comes under actuation since the point of application of voltage shifts to the disc in leg 6. The combination of these two effects, results in total loss of force in legs 1 and 5 very quickly due to the geometric reasons mentioned earlier. It will also result in a relatively sharp initial increase in the loads on legs 2 and 6 (from no force initially) as the discs in these two legs make contact with the respective legs. Subsequently there will be a steady increase in the force to a maximum value as the second loading step advances.

This loading scenario is captured by the loading sequence depicted in Figure 14. At the beginning of the second loading step, the points of actuation in legs 2 and 6 have a non-zero displacement. This displacement of the points of actuation decreases as the second loading step proceeds, reaching zero displacement at some point during the loading step. The exact nature of this complicated process is hard to determine and as an idealization, it is assumed that the displacements of the points of actuation decrease linearly to a value of zero at the end of the step. It is however expected that this idealization will still be able to capture, qualitatively, the response of the skin to a series of loadings.

To summarize, in any single loading step the force on the two corresponding pairs of legs that are undergoing loading increases linearly from zero to the maximum value while the deflections of the points of application of the force decrease linearly to zero from the value that they held at the end of the previous loading step.

At the end of the loading step, the force acting on the legs immediately drops to zero and the boundary conditions on the legs are removed. The force on the subsequent pair of legs now starts increasing linearly from zero to reach its maximum value while the displacements of these new points of application of the forces reduce linearly to zero at the end of this new step. The same process goes on as a sequence as the traveling wave proceeds.

8.1.1. Numerical Implementation. The Newmark Scheme with $\alpha = 0.45$ and $\gamma = 0.45125$ is used for time integration. This scheme is conditionally stable \([31]\) for the above values of $\alpha$ and $\gamma$ and additionally results in a certain amount of numerical damping that damps out high frequency noise. To ensure accuracy and stability of the solution, the adaptive time incrementation capability of ABAQUS is utilized. This adaptive time incrementing scheme works on the basis of maintaining the maximum half-step residual occurring in the FE model within certain user specified limits. The value of the half-step residual should be moderately small compared to the significant forces in the problem ($\pm 1$N in this case)
to ensure a high degree of accuracy. By testing the accuracy and stability for some sample cases with successively smaller sizes of time incrementation (without utilizing the adaptive time incrementation capability) the half-step residual that would ensure accuracy and stability in a multi-step dynamic analysis was found to be 0.01N. The adaptive time incrementation scheme is critical in resolving the transient effects that dominate the response during the change from one loading step to another, while still maintaining the computational costs within reasonable limits. The finite element mesh used in the dynamic analysis is a sufficiently refined mesh, the accuracy of which has been independently verified by performing static analysis with successively refined meshes.

8.1.2. Rayleigh Damping. To model the combined effect of the inertial and viscous forces due to the fluid on the skin as well as the intrinsic material damping present in the skin, Rayleigh damping ([32]) or Proportional damping is used. Since the damping matrix, as derived in a finite element formulation, is not purely an element property and is generally frequency dependent, it is difficult to assemble a global damping matrix from element damping matrices. As a result, the damping matrix is usually constructed as a linear combination of the assembled mass and stiffness matrices as given below:

$$
C = aK + bM
$$

Where, $K$, $C$ and $M$ are the stiffness, damping and mass matrices, respectively. This formulation decouples the set of ordinary (coupled) differential equations derived from the finite element formulation and provides the following equation, relating $a$ and $b$ to experimentally observable parameters:

$$
a + b\omega_i^2 = 2\omega_i \xi_i
$$

$$
\omega_i = 2\pi f_i
$$

Here $f_i$ is the $i^{th}$ natural frequency of the skin and $\xi_i$ is the damping ratio corresponding to $f_i$. The implicit assumption in this formulation is that the total damping experienced by the skin is the sum of individual contributions from each of the eigenmodes. By experimentally observing the damping ratios and the corresponding natural frequencies in the vibrations of structures that are similar in geometry to the one being studied (and made of the same material), the values of $a$ and $b$ can be determined. Rayleigh damping therefore approximates overall energy dissipation (per cycle) in the system response, by constructing a damping matrix that matches the energy dissipation (per cycle) observed in real life in a similar system.

From the existing literature ([33]) on experimental observations of vibrations of an aluminum cantilever beam in air, the values of the damping ratios and the natural frequencies for the first two modes of vibration are obtained as $\xi_1 = 0.01371$, $\xi_2 = 0.00411$, $f_1 = 60Hz$ and $f_2 = 365Hz$. Using these experimental observations, the values of $a$ and $b$ are calculated as $a = 10.1$ and $b = 1.664 \times 10^{-6}$. Using equation (12), it can be seen that this formulation, in effect, allocates damping ratios of 0.0246 and 0.0555 for the first two eigenmodes of the active skin.

8.1.3. Results from Dynamic Analysis. Multi-step dynamic analyses were performed for four cases with the applied load increasing from 0 to +/-1N in each loading step, in a time interval of $T = 20ms$, 0.5ms, 0.2128ms and 0.125ms. These correspond to actuation frequencies of 50Hz, 2kHz, 4.7kHz (first natural frequency of the skin: condition of resonance) and 8kHz, respectively. Here, the term actuation frequency refers only to the frequency with which the individual loading steps take place. Since eight loading steps are required for the wave pattern to "travel" by one wavelength, the frequencies of the traveling wave for these respective scenarios are one eighth the actuation frequencies. Figure 15 shows the displacement of a node on the top surface of the skin, directly above the center of leg 3 (as shown in Figure 13). The total duration of analysis involved twelve loading steps, which amounts to one and a half times the period of the wave.
While the displacement plot for an actuation frequency of 50Hz (Figure 15) appears linear, a closer inspection (Figure 16) reveals that dynamic effects do exist. Furthermore, the frequency of the repeated pattern corresponds to the first eigenfrequency (4.7kHz), indicating the predominance of the first eigenmode in the response of the skin at these actuation frequencies. However, the magnitude of these dynamic effects is very low at this frequency. The effect of the damping forces introduced by Rayleigh damping combined with the small magnitude dynamic effects, result in an overall deflection pattern that practically matches that obtained from a quasi-static analysis. Further evidence for this observation is available from the velocity plots of the same node under consideration (Figure 17 and Figure 18).
The overall velocity plot (Figure 17) indicates that, at the beginning of a new loading step, strong transient effects do exist as a result of the sudden change in loading. However, these velocity fluctuations die down quickly and the velocity reaches a constant value as the loading step proceeds. Figure 18 is a closer look at the velocity fluctuations right at the beginning of the first loading step. The frequency of the oscillations again corresponds to the first eigenmode (4.7kHz). The decrement of the velocity amplitudes also corresponds to the damping ratio allotted to the first eigenmode, through the method of Rayleigh damping. The dynamic effects, however, become more evident at higher actuation frequencies, and the response is dominated by the first eigenmode.
Figure 19. Displacement plot of the node above leg 3 for an actuation frequency of 2kHz, plotted over twelve loading steps, which corresponds to one and a half times the period of the wave.

Figure 20. Displacement plot of the node above leg 3 for an actuation frequency of 4.7kHz (Resonance Condition), plotted over twelve loading steps, which corresponds to one and a half times the period of the wave.

The node corresponding to maximum/minimum deflection for any given loading step also corresponds to the position of maximum/minimum deviation (“antinode”) for the first eigenmode. The contribution from the first eigenmode to the nodal deflection thus becomes significant at the instants of time when the node corresponds to the position of maximum or minimum deflection for that loading step. As a result, the dynamic effects would be most evident when the nodal deflection reaches its maxima/minima.

This is clearly seen both in Figure 19 and in Figure 20, which correspond to actuation at frequencies of 2kHz and 4.7kHz, respectively. The superimposed oscillations in the nodal displacement, as the nodal displacement reaches a maxima/minima, occur at the first eigenfrequency, clearly indicating the strong contribution from the first eigenmode at these instants of time. This effect is most pronounced in the latter case because the actuation is taking place with a single step time period that corresponds to the first natural frequency (Resonance).
However, this effect is absent in Figure 21 since the actuation frequency is higher than the first natural frequency of the skin. Since the time scales involved are so small at this actuation frequency, the higher eigenmodes contribute significantly to the response, before they die down due to Rayleigh damping, resulting in a smooth displacement profile.

Figure 21. Displacement plot of the node above leg 3 for an actuation frequency of 8kHz, plotted over twelve loading steps, which corresponds to one and a half times the period of the wave.

However, in all four cases studied, the deflection amplitude observed is quite close to the value obtained through a quasistatic analysis. There is only a slight increase in the deflection amplitude with an increase in the actuation frequency. This is due to the Rayleigh damping introduced in the numerical simulations and is evident in Figure 22. In the absence of Rayleigh damping the deflection pattern grows continuously with time when the actuation frequency matches the first natural frequency, setting up resonance. The growing superimposed oscillations in the undamped response are an indication of the growth of the contribution from the first eigenmode under this resonance condition. In comparison, the damped response is clearly periodic and stable.

Figure 22. Undamped vs. damped response for actuation at the first natural frequency.

Figure 23 shows the displacement profiles of three nodes numbered 1, 2 and 3 that lie on the top surface of the skin directly above the center of legs 1, 2 and 3, respectively, for actuation at a frequency of 8kHz.
It is evident that the displacement pattern experienced by every node is identical, periodic and shifted by a constant phase difference. This is precisely the description of a traveling wave. Another point to be noted is that the transients introduced in the first few loading steps of the actuation sequence die down within one period of the traveling wave. The response of the skin then steadies into a regular periodic pattern. This is an important observation indicating that the response of the skin neither grows nor decreases with time.

8.2. Piezoceramic Stack Actuator Based Design

The skin dimensions and the skin material used for the dynamic analysis of the Piezoceramic stack actuator based design are identical to those used in the earlier analysis ($L=25\text{mm}$, $h=0.5\text{mm}$ and no legs; skin made of aluminum: $E=70\text{GPa}$, $\nu=0.345$ and $\rho=2600\text{ kg/m}$). However the force amplitude used here is $0.31\text{N}$ which is the force that generates a quasistatic deflection amplitude of $30\text{ µm}$ in this design. Once again the same periodicity condition is invoked as in the earlier section and the mesh density and the numerical recipes used are all identical to the ones used in the earlier section. As in the quasistatic analysis, the periodic element is loaded at eight equally spaced intervals. All the loads vary sinusoidally with time, have an amplitude of $0.31\text{N}$, and each successive load lags the previous load by a phase difference of $45^\circ$. The analysis was performed for four different actuation frequencies: 1Hz (nearly quasistatic), 4kHz, 8kHz (resonance) and 10kHz. The stabilized response of one point on the skin is plotted over one non-dimensionalized time period in Figure 24, for three actuation frequencies (1Hz, 4kHz and 10 kHz). For the 1Hz case, the deflection amplitude of the response is $39.92\text{ µm}$ whereas the 10kHz case it is $52.25\text{ µm}$. In all these cases, the responses stabilized within a short duration of time. Figure 25 plots the response of a point for the case wherein the loading frequency is the $1^{st}$ natural frequency. This plot clearly indicates that response is in resonance. The stabilized deflection amplitude of this response is $1.157\text{ mm}$, which is almost two orders of magnitude greater than the response for the other cases. However, even in this case, the response saturates in about 15 cycles. The Rayleigh Damping parameters used in these analyses are identical to those used in the earlier section for the analysis of the SMA actuated design. This only goes to show that the system is not overdamped in either set of analyses. The sharp difference in response for the resonance case between the SMA and the Piezostack designs can be explained by the fact that the loading in the latter much more closely resembles the first eigen mode of the skin than the former. Again, the response of any two points on the skin are identical in shape and merely phase shifted, the amount of phase shift depending on the distance between the two points.

To conclude, the dynamic analysis of the Piezoceramic Stack Actuator based design clearly indicates that the response of the skin (even at high/resonant frequencies) tends to be self similar to the imposed loading pattern. Moreover the significant amplification produced during resonance suggests that the actuation requirements could be further reduced by designing the skin to have a natural frequency close to the operational frequencies for that application.
Comparison of Skin Responses for Different Actuation Frequencies

Figure 24. Stabilized response of a point on the skin for actuation at frequencies, 1Hz, 4kHz and 10 kHz

Figure 25. Saturation of response of a point on the skin for actuation at first natural frequency
10. CONCLUSIONS

A deformable active skin actuated by active materials was proposed for an active flow control technique that holds promise for large reduction in turbulent skin friction drag. A generalized actuation principle was developed that is capable of generating a traveling sine wave on the surface of the active skin. The closed form solution for this generalized loading was obtained and it was seen that if the loading function is a traveling wave in the form of a harmonic, then the response would be self-similar to the loading, but with a phase lag. Any practical realization of the active skin would, however, employ a discrete number of actuators, which will allow only for an approximate implementation of this generalized actuation principle.

Two actuation schemes, a force based and a moment based scheme, that utilize a discrete number of actuators, and hence approximations of the generalized actuation principle, were proposed. Taking advantage of the periodicity in the deflections, theoretical analyses of the two actuation schemes were performed by reducing the domain of analysis to a periodic element. On account of the difficulty in performing theoretical analysis of these two schemes in a dynamic setting, the theoretical analyses were restricted to quasistatic and eigenvalue analysis. Upon comparison, it was found that the work efficiency of the force based actuation scheme was higher than that of the moment based actuation scheme from a structural point of view.

Three different skin designs (that implement either of the two actuation schemes) were proposed that utilize SMA, piezoelectric C-block and piezoceramic stack actuators for actuation, respectively. The theoretical results obtained for the actuation schemes, were refined by performing FEA on the actual designs, and the impact of the presence of “legs” on the deflection amplitudes and natural frequencies were studied. It was generally found that the presence of “legs” while moderately increasing stiffness (and thus reducing deflection amplitude), significantly lowered the natural frequencies by substantially increasing the inertia of the system.

Micro scale actuation requirements translate into small temperature differentials and small strains (<0.6%) for SMA based actuation, which allows for relatively high actuation frequencies (order of 50 Hz). However, cooling of SMAs is the biggest challenge to overcome for SMA based implementations of the active skin. Piezoelectric C-block actuator and Piezoceramic stack actuator based designs are feasible for the force and stroke requirements for the active skin implementation. However, issues relating to geometric constraints need to be resolved for successful implementation of piezoelectric actuation based designs. Motion amplification systems are one solution, but they would increase design complexity.

Multi-step dynamic analysis validated the efficacy of the SMA based active skin design in generating a traveling wave profile. Transient dynamic effects in the skin play a minor role due to the presence of damping in the system. A steady state periodic response from the skin is attainable even at high actuation frequencies. The response of the skin when actuated at its first natural frequency is primarily dominated by the first eigenmode. However, the damping introduced into the system in the form of Rayleigh damping ensures that the response of the skin stays bound with time. Deflection amplitudes were found to be close to the quasistatic results, with the deflection amplitudes increasing only slightly with increasing actuation frequency.

Dynamic analysis of the Piezoceramic stack actuator based design revealed that even at high actuation frequencies, the stabilized response of the skin resembles the original loading function and hence is a good approximation of a traveling sinusoidal wave. Actuation at the first natural frequency results in a very large amplification of the response (almost 2 orders of magnitude greater than the non-resonant cases), which stabilizes quickly. This indicates that there exists the potential for further reduction in actuation forces by tailoring the skin designs such that the natural frequency of the skin matches the operational frequencies for that application.

It can be generally summarized that for applications in which the required actuation frequencies are low (order of 50 Hz for ROVs), the SMA based actuation technique holds promise, whereas for applications involving high actuation frequencies (several hundred Hz, for airliners and military aircraft applications), the piezoelectric actuator based systems would be more appropriate.

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