

IMPACT INDUCED PHASE TRANSFORMATION IN SHAPE MEMORY ALLOYS

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ABSTRACT

A dynamic analysis is given of the impact induced phase transformation in a shape-memory alloy rod, with a special focus on the propagation of stress waves and phase transformation fronts in the rod. The material behavior of the shape-memory alloy is modelled by a thermomechanical constitutive theory developed by Lagoudas, Bo, and Qidwai (1996), which is based on the formulation of Gibbs free energy that depends on, among other variables, the martensitic volume fraction and the transformation strain, along with evolution equations derived from a dissipation potential theory. Field equations and jump conditions of the fully coupled thermal-mechanical problem are derived to account for balances of linear momentum and energy. The equations are solved using the method of characteristic curves. The solutions are found to be associated with shocks, across which various field quantities suffer jump discontinuities. A typical solution involves two wave fronts which are initiated at the impact surface and propagate into the rod. One, travelling at the acoustic speed, separates the tranquil and disturbed regions. The other, travelling at a lower speed, separates the regions of the martensitic and austenitic phases. It is found that the stress and temperature jumps across the phase boundary can be significant. A numerical example is presented.

Keywords. A: Dynamics; Phase transformation; Shock waves; B: Polycrystalline material; C: Characteristics

1. INTRODUCTION

Due to their unique thermal and mechanical properties, shape memory alloy (SMA) materials have found numerous applications in mechanical, electronic and automotive

engineering, aerospace industry, domestic appliances, and medical devices. (See, for example, the review article by Birman (1997).) Many applications, such as SMA actuators in dynamic environments, require an understanding of the response of SMAs to dynamic loads. The problem of impact induced phase transformation in SMAs provides an excellent setting in gaining this understanding by studying the generation of phase transformations at the impact surface and the propagation of phase transformation regions in the SMA material, as well as the relations among mechanical/thermal fields, material properties and loading conditions.

A sound constitutive theory is the starting point of a successful analytical modeling, either dynamic or static. A number of constitutive models have been proposed for SMAs, and in general for materials capable of undergoing phase transformations. Some of these models are reviewed in Birman (1997).

Abeyaratne and Knowles (1993), and Abeyaratne, Kim and Knowles (1994) developed a one-dimensional thermoelastic constitutive model based on a Helmholtz free-energy function, a kinetic relation and a nucleation criterion. The free energy is associated with a multi-well potential energy function, the kinetic relation is based on thermal activation theory, and nucleation is assumed to occur at a critical value of the appropriate energy barrier. Under certain loads, the material is partitioned into domains associated with different phase wells, that are separated by sharp phase boundaries. Their constitutive theory appears to be particularly suitable to model SMA single crystals.

Various constitutive theories have been proposed based on micromechanical considerations in attempt to model polycrystalline SMAs. These models involve internal state variables which characterize the extent of phase transformation, as well as evolution equations which relate the internal variables to other state variables. Among others, Tanaka and Nagaki (1982) derived three-dimensional incremental constitutive equations that relate the time derivative of stress tensor to those of strain tensor, temperature, and the martensitic volume fraction. Tanaka, Kobayashi and Sato (1986), and Sato and Tanaka (1988) derived an evolution equation that expresses the time derivative of the martensitic volume fraction in terms of the state variables. A different form of evolution equation was proposed by Liang and Rogers (1990) and Liang (1990). Liang and Rogers (1991) also developed a three-dimensional theory involving an equivalent strain, based on

the assumption that the phase transformations are governed by the distortional energy.

To account for simultaneous phase transformation and reorientation that occur in polycrystalline SMAs under nonproportional loading or in composites with SMA components, Boyd and Lagoudas (1993, 1994, 1996) developed a three-dimensional theory in which the thermoelastic coefficients depend on the martensitic volume fraction according to the rule of mixtures, and the martensitic volume fraction depends on the effective stress calculated from the deviatoric stress tensor. This model has been applied to the analysis of metal matrix composites by Lagoudas, Bo and Qidwai (1996), and yielded solutions in a qualitative agreement with experimental data.

There have been fairly extensive analytical, experimental and numerical investigations of static and quasi-static behaviors of SMAs. On the other hand, the existing study of dynamic behaviors of SMAs is rather limited, despite a clear need for understanding. The only experimental investigation that we were able to find in the literature concerning dynamic behaviors of SMA under impact loads was given by Escobar and Clifton (1993). They carried out pressure-shear plate impact experiments on Cu-14.44Al-4.19Ni single crystals to study the kinetics of the stress-induced phase transformation. Measured velocity profiles provide several indications of the existence of a propagating phase boundary. These include a difference between the measured particle velocity and the transverse component of the projectile velocity, and the evidence that the phase boundary causes a reflected quasi-longitudinal wave and a diffracted quasi-shear wave which arrive at the rear surface of the target at expected times. By the experimental data obtained in the above work, Abeyaratne and Knowles (1997) determined, using the constitutive model they developed, the values of phase boundary velocity and driving force, as well as the kinetic law which relates these two quantities.

In a general dynamic setting, impact loading on a solid material that admits phase transformations may give rise to both conventional shock waves and traveling regions within which phase transformations are taking place. Pence (1986) gave a dynamic analysis of a semi-infinite elastic bar whose stress-strain relation is not monotonic, subjected to a monotonically increasing end load. He found that if the load is sufficiently high, a strain discontinuity emerges at the end of the bar and subsequently propagates into the interior. For long periods of time, the phase boundary velocity approaches a con-

stant. Abeyaratne and Knowles (1991) studied the dynamics of phase transformations in elastic bars with a piecewise-linear and non-monotone stress-strain relation, along with two additional constitutive requirements: a kinetic relation controlling the rate at which the phase transformation takes place and a nucleation criterion for the initiation of the phase transformation. They found that the kinetic relation and the nucleation criterion together single out a unique solution from among the infinitely many solutions that satisfy the entropy jump condition at all strain discontinuities. In a related problem, Pence (1991) studied the encounter of an acoustic shear pulse with a phase boundary in an elastic material with non-monotone stress-strain relation. In particular, he showed that a pre-existing stationary phase boundary can be set in motion by a finite amplitude shear pulse. Abeyaratne and Knowles (1994a, 1994b) extended their earlier dynamic analyses by including thermal effects. They considered the initial value problems for infinite rods in which the initial data involves two distinct phases. Both heat conduction and adiabatic cases are studied.

In the above studies, the materials are characterized by multi-well potential energy functions. Different phases of the material are associated with these wells. The regions of different phases are divided by sharp phase boundaries that are discontinuities in deformation gradient. The velocity of a phase boundary is directly related to a certain driving traction through a kinetic relation which is taken as a constitutive relation of the material. Often, certain assumptions are made to simplify the analysis.

In this paper, we study the impact problem for SMA materials using a constitutive theory developed for polycrystals. A one-dimensional problem is considered in which a semi-infinite rod is subjected to an end impact load of prescribed constant stress. One of the focuses is on the generation and propagation of phase transformation regions from the impact end and on how the structure of such transformation regions depends on the impact stress. The full thermo-mechanical coupling effect is considered, with attention being paid to both the stress and temperature distributions in the transformation regions.

In Section 2, we formulate the dynamic problem using the balance laws and the entropy inequality. Both field equations and jump conditions are derived. The constitutive theory developed by Lagoudas, Bo and Qidwai (1996) is used, which involves, for the present one-dimensional problem, 2 internal variables: the martensitic volume fraction

