3-D FEA Modeling of Ni60Ti40 SMA Beams as Incorporated in Active Chevrons

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• All FEA analysis performed using **ABAQUS research license**.
Overview

• **Introduction** to SMAs
• **Motivation** for development of numerical analysis tools
• **Analysis Example: Introduction** of Boeing VGC
• “Unified Model”
  – Original Form
  – Improvements developed/implemented
• **Analysis Example: Results**
  – Experimental characterization
  – Calibration and validation of model
  – Analysis results
• **Conclusions**
Motivation: Development of Powerful Numerical Analysis Tools (1/2)

• More advanced smart structures incorporating active materials being considered (complex/large deformation, inhomogeneous stress, etc)

• Complex structures require more powerful analysis tools
Motivation: Development of Powerful Numerical Analysis Tools (2/2)

Legacy Method: Design, Build, Test, Iterate → Optimize

Preferred Method: Characterize, Analyze → Optimize
Analysis Example: The Boeing VGC

Calkins, Mabe, & Butler, SPIE, 2006

Mabe, Calkins, & Butler, 47th AIAA, 2006
The Boeing VGC

Analysis Problem

Determine the mechanical response of the active chevron provided given temperature changes to the SMA beam elements.

Focus on chevron deflection, especially with regard to the free stream.
Analysis Example: Ni60Ti40

- **Ni60-Ti (wt %) = Ni55-Ti (at %)**
- Boeing chevrons pioneered use in aerospace applications
- Nickel rich → additional precipitates
- Precipitates lead to the following attributes:
  - Thermomechanical stability
  - Transformation temperatures set by heat treatments
  - No initial cold work required to promote the shape memory effect → complex shapes

Mabe, Ruggeri, & Calkins, Int’l Conf Shape Memory & Superelast., 2006; Clingman, Calkins, & Smith, SPIE, 2003
Introduction – SMA Behavior

SMA Phase Diagram (Schematic)

- Stress, $\sigma$
- Temperature, $T$

1. Martensite (Detwinned)
2. Martensite (Twinned)
3. Austenite

- $M_f$, $M_s$, $A_s$, $A_f$
- $C^M$, $C^A$
- $\sigma^t$, $\sigma^s$
- $M'^t$ to $M^d$, $A$ to $M^d$, $M^d$ to $A$

Stress-strain curves for different phases:

- 1: Martensite to Austenite
- 2: Austenite to Martensite
- 3: Martensite transition

Graphical representation showing the phase transitions and stress-strain behavior for shape memory alloys.
Thermomechanical Characterization (1/2)

I. Plates of various thickness received
II. ASTM “subsized” dogbone specimens prepared
III. Thermomechanical loading paths applied

4.3mm thick
Characterization (2/2)

- **Constant Stress**
  - 300 MPa
  - 250 MPa
  - 200 MPa
  - 150 MPa
  - 120 MPa
  - 90 MPa

- **Applied Stress Test Level (MPa)**
  - 0
  - 50
  - 100
  - 150
  - 200
  - 250
  - 300
  - 350
  - 400

- **Temperature (°C)**
  - -30
  - 20
  - 70

- **Max. Trans. Strain**
  - 0.0%
  - 0.4%
  - 0.8%
  - 1.2%
  - 1.6%

- **Stress (MPa)**
  - 0
  - 50
  - 100
  - 150
  - 200
  - 250
  - 300
  - 350
  - 400

- **Austenite**
  - \( M_f \)
  - \( M_s \)
  - \( A_s \)
  - \( A_f \)

- **Martensite**
  - \( C^M_{\sigma=300} \)
  - \( C^A_{\sigma=300} \)
Unified Model: Original Formulation

Gibbs Free Energy:

\[ G(\sigma, T, \varepsilon', \xi) = -\frac{1}{2\rho} \sigma : S : \sigma - \frac{1}{\rho} \sigma : \alpha \Delta T \]
\[ + c \left[ \Delta T - T \ln \left( \frac{T}{T_0} \right) \right] - s_0 \Delta T + u_0 + f(\xi) \]

Transformation Surfaces:

\[ \xi \geq 0, \quad \Phi \leq 0, \quad \Phi \xi = 0 \]
\[ \xi \leq 0, \quad \Phi \leq 0, \quad \Phi \xi = 0 \]

Evolution Equation:

\[ \dot{\varepsilon}' = \Lambda \xi \]

Hardening Function:

- Exponential (Sato & Tanaka, 1986)
- Polynomial (Boyd & Lagoudas, 1995)
- Cosine (Liang & Rogers, 1990)
More General Evolution Equation Implemented

Original:

\[ \dot{\xi} = \Lambda \dot{\xi} \]

\[
\Lambda = \begin{cases} 
\frac{3}{2} H \frac{\sigma'}{\bar{\sigma}}, & \xi > 0 \\
H \frac{\varepsilon^{t-r}}{\bar{\varepsilon}^{t-r}}, & \xi < 0 
\end{cases}
\]

Generalized: (Bo/Lagoudas, 1999)

\[
\Lambda = \begin{cases} 
\frac{3}{2} H_{\text{cur}} (\bar{\sigma}) \frac{\sigma'}{\bar{\sigma}}, & \xi > 0 \\
\frac{\varepsilon^{t-r}}{\xi}, & \xi < 0 
\end{cases}
\]
New “Smooth”
Hardening Function Developed (1/3)

- Same thermodynamic framework of Unified Model
- Same implementation scheme – Return Mapping Algorithm
- New Hardening function - Continuous function with continuous derivatives
  → Smooth transitions between elastic phases and phase transformation

Original Form: (Polynomial)

\[
\frac{df}{d\xi} = \begin{cases} 
    b_1\xi + b_2; & \xi > 0 \\
    b_3\xi + b_4; & \xi < 0 
\end{cases}
\]

New Form: (Smooth)

\[
\frac{df}{d\xi} = \begin{cases} 
    \frac{1}{2}a_1 \left(1 + \xi^{n_1} - (1 - \xi)^{n_2}\right); & \xi > 0 \\
    \frac{1}{2}a_2 \left(1 + \xi^{n_3} - (1 - \xi)^{n_4}\right); & \xi < 0 
\end{cases}
\]
New “Smooth” Hardening Function (2/3)

Conformity with Experimental Results - Pseudoelasticity

Polynomial

“Smooth” Form:

(Trained NiTi wire, T=303K)
New “Smooth” Hardening Function (3/3)

Conformity with Experimental Results - DSC
Simulation of Experiments: Influence of Hardening Function

1-D Loading of 3-D BVP to validate material parameters chosen

Polynomial Hardening Function

Model/Experiment Matching
Assembly
- SLOT connectors “bolt” down SMA beams
- SLIDE-PLANE connectors prevent beam rotation
- Contact enforced: SMA beams and chevron (no friction)

Loading Steps
1. Clamp beams \((T<A_s)\)
2. Heat beams \((T>A_f)\)
3. Cool \((M_f<T<M_s)\)
4. Heat beams \((T>A_f)\)
Results of Analysis (1/2)

Stress (VM) Contours

Deflection Contours

Centerline Profile

Tip Deflection History
Results of Analysis (2/2)

Comparison of flight test data with analysis; Take-off condition

(Calkins, Butler, Mabe: AIAA 2006-2546)
Conclusions

• Original “Unified Model” implementation has been augmented to include new material effects
  — More general evolution equation
  — New hardening function simulating more “smooth” material response

• Improved Unified Model used to analyze complex aerostructure

• Current and Future work:
  — Addition of permanent plastic yield surface
  — Detailed validation
  — Extension to other applications
Current Work: Plastic Yield Surface

![Diagram showing plastic yield surface with stress-strain curves and temperature-T stress phase transformation graph.](image-url)
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Model Calibration

### Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^A$</td>
<td>90GPA</td>
</tr>
<tr>
<td>$E^M$</td>
<td>47GPa</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.33</td>
</tr>
<tr>
<td>$\alpha^A$, $\alpha^M$</td>
<td>10.0E-6/ºC</td>
</tr>
<tr>
<td>$H_{\text{cur}}(\sigma)$</td>
<td>$=0.0135[1-e^{-0.008*\sigma_{VM}}]$</td>
</tr>
<tr>
<td>$M_s$</td>
<td>34ºC</td>
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<tr>
<td>$M_f$</td>
<td>-17ºC</td>
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<tr>
<td>$A_s$</td>
<td>23ºC</td>
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<tr>
<td>$A_f$</td>
<td>57ºC</td>
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<tr>
<td>$\rho \Delta s^A_0$</td>
<td>-0.300MPa /ºC</td>
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<tr>
<td>($C^A</td>
<td>_{\sigma=300}$</td>
</tr>
<tr>
<td>$\rho \Delta s^M_0$</td>
<td>-0.212MPa /ºC</td>
</tr>
<tr>
<td>($C^M</td>
<td>_{\sigma=300}$</td>
</tr>
</tbody>
</table>
Phase Diagram for “Smooth” Hardening Function

Temperature (°C)

Stress (MPa)

Martensite

Austenite